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ON THE THEORY OF CURRENT AND VOLTAGE RESONANCE IN AN RLC-OSCILLATING CIRCUIT

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Аннотация:
данная статья посвящена резонансам тока и напряжений в RLC-колебательных цепях. Особое внимание уделено теоретическим основам резонансных процессов.

Ключевые слова: резонанс, колебательный контур, резонанс заряда, резонанс тока, резонанс напряжений, добротность, дифференциальное уравнение, вынуждающая сила, затухающее колебание, коэффициент затухания, векторная диаграмма.

Abstract:
This article is devoted to current and voltage resonances in RLC oscillatory circuits. Particular attention is paid to the theoretical foundations of resonant processes.

Keywords: resonance, oscillatory circuit, charge resonance, current resonance, voltage resonance, quality factor, differential equation, driving force, damped oscillation, damping coefficient, vector diagram.

If the value of the oscillatory circuit is large during the formation, or more precisely, during the generation of electromagnetic oscillations and waves, then the value of the phenomenon of resonance in the oscillatory circuit is infinite when transmitting electromagnetic waves into space or capturing and using electromagnetic waves. This is the basis of the principle of radio transmission and reception [1]. Based on this, thanks to this article we will get acquainted with the theory of current and voltage resonances in RLC oscillatory circuits.

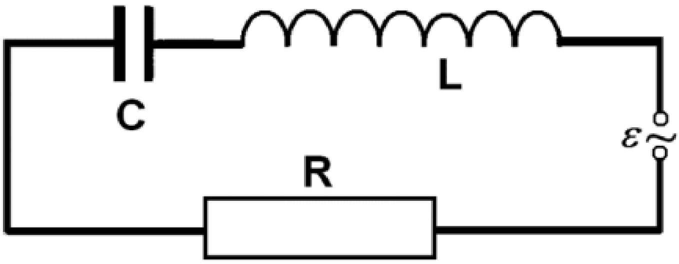

Fig.1. RLC oscillatory circuit diagram

Based on Figure 1, we write Kirchhoff's second law for this circuit to consider the theory of the process occurring in an RLC oscillatory circuit under the influence of a variable EMF: $e=e_o\cos\omega t$:

$$R \cdot I + U_C = e - L\frac{dI}{dt} \quad (1)$$

Here UC=q/C is the voltage drop across the capacitor, I=dq/dt is the current in the circuit. Let us write equation (1) in the form of a differential equation:

$$L\frac{d^2q}{dt^2}+R\cdot\frac{dq}{dt}+\frac{q}{C} = e_o\cos\omega t \quad (2)$$

Dividing all terms of the equation by L, we obtain a differential equation with a non-uniform constant coefficient for charge q:

$$\ddot{q} + \frac{R}{L}\cdot\dot{q} + \frac{q}{LC} = \frac{e_o}{L}\cos\omega t \quad (3)$$

From here we obtain a differential equation in which both damping and driving forces participate:

$$\ddot{q} + 2\beta\dot{q} + \omega_o^2 q = f_o\cos\omega t \quad (4)$$

where $\beta=R/2L$, $\omega_o=\sqrt{(1/LC)}$, $f_o=e_o/L$, as we know, mass is a measure of inertia in mechanical processes, then in electromagnetic processes the measure of inertia is inductance and the inverse value of capacitance plays the role of an elasticity coefficient in spring pendulum.

The general solution of such a differential equation is sought in the form of the sum of the general solution of the homogeneous equation x_1 and the particular solution of the inhomogeneous equation x_2

$$x = x_{1,general\ homogeneous} + x_{2partial\ heterogeneous,};$$

$$x = x_{1,,the\ general\ is\ more\ homogeneous} + x_{2,damped\ oscillation} = A_1 e^{-\beta t}\sin(\omega_1 t + \alpha_1);$$

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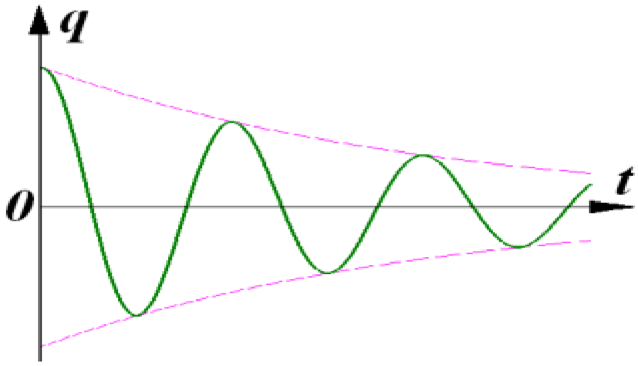
In the absence of a compelling force, i.e., in the absence of a variable ECU, the expression of the frequency of the extinguishing oscillation is as follows:

$$\omega_1 = \sqrt{\omega_o^2 - \beta^2} = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

The frequency of free oscillations of the system in the absence of a driving force, and in the case of an RLC circuit - in the absence of an alternating EMF:



Rice. 2. Graph of free electromagnetic oscillation

The solution x_2 determines in mechanics the nature of the motion of the system, performing steady-state forced oscillations, and for the oscillatory circuit x_2 describes the law of charge changes on the capacitor plates subject to the action of an EMF in the oscillatory circuit: $e = e_o cos\omega t$[3].

To find $x_2 = A_2 \cos(\omega t - \varphi_2)$ we use a vector diagram as this was done when studying mechanical vibrations:

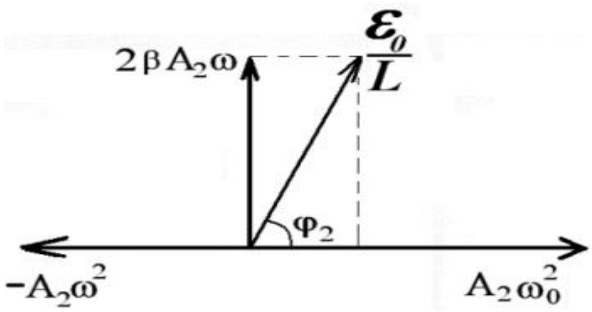


Fig.3. Vector diagram

From the vector diagram we find A_2:

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$$A_2 = \frac{f_o}{\sqrt{(\omega_o^2)^2 - \omega^2 + 4\beta^2\omega^2}} = \frac{e_o/L}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + 4\left(\frac{R}{2L}\right)^2 \omega^2}}$$

Here ω is the frequency of the EMF variable.

$tg\varphi_2 = \frac{2\beta\omega}{\omega_o^2 - \omega^2} = \frac{R\omega}{L\left(\frac{1}{LC} - \omega^2\right)} = \frac{R}{\frac{1}{\omega C} - \omega L}$ the sign of $tg\varphi_2$ can be negative if $\omega_o < \omega$ and, therefore, in this case the phase shift between the driving force and the displacement is $\varphi_2 > \pi/2$.

$$x_2 = \frac{e_o/L}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{R}{L}\right)^2 \omega^2}} \cos(\omega t - \varphi_2) = q(t)$$

This solution corresponds to the law of changes in charge on the capacitor plates and voltage on its plates:

$$U_C = \frac{e_o/LC}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{R}{L}\right)^2 \omega^2}} \cos(\omega t - \varphi_2).$$

Let us examine the resulting solution[4].

If the active resistance of the circuit R tends to 0, then the attenuation coefficient of the system is:

$$\beta \to 0 \text{ at the same time } q = A_1 \sin(\omega_o t + \alpha_1) + \frac{e_o/L}{\frac{1}{LC} - \omega^2} cos\omega t$$

In this case, the first term does not decay, A_1 and α_1 are determined from the initial conditions.

At $\beta \to 0 \quad A_2(\omega \to \omega_o) \to \infty$.

1) Find the current in the circuit:

$$I = \frac{dq}{dt} = -\frac{\frac{\omega e_o}{L}}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{R}{L}\right)^2 \omega^2}} \sin(\omega t - \varphi_2).$$

The current in the circuit, and therefore the voltage across the active resistance, is phase shifted by π/2 relative to the voltage across the capacitor and, as will be shown below, by (-π/2) relative to the voltage across the inductance [5].

Note that the phase shift of the current relative to the EMF phase is determined by the angle $\varphi = \varphi_2 - \frac{\pi}{2}$:

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$$I=\frac{dq}{dt}=\frac{\frac{\omega e_o}{L}}{\sqrt{\left(\frac{1}{LC}-\omega^2\right)^2+\left(\frac{R}{L}\right)^2\omega^2}}\cos\left(\omega t-\varphi_2+\frac{\pi}{2}\right)=\frac{\frac{\omega e_o}{L}}{\sqrt{\left(\frac{1}{LC}-\omega^2\right)^2+\left(\frac{R}{L}\right)^2\omega^2}}\cos(\omega t-\varphi)$$

At the same time tg$\varphi=\frac{1}{R}(\omega L-\frac{1}{\omega C})$.

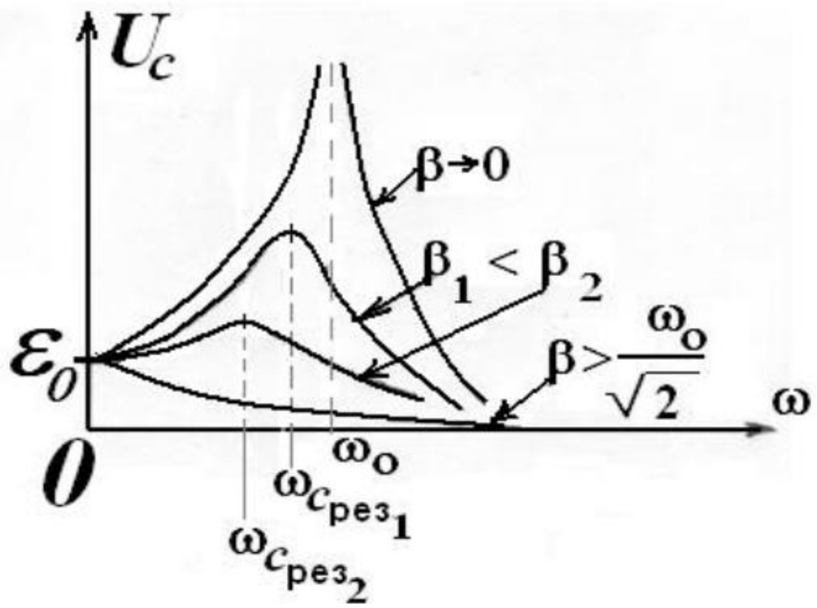
Resonance in an oscillatory circuit. Let's find the maximum of $A_2(\omega)$ when the denominator is minimal:

$$A_2=\frac{f_o}{\sqrt{(\omega_o^2)^2-\omega^2+4\beta^2\omega^2}};$$

From this comes $\omega_{rez}=\sqrt{(\omega_o^2-2\beta^2)}$ - the resonant frequency-frequency at which an EMF with amplitude e_o can excite voltage and charge oscillations on the capacitor plates with maximum amplitude.

At $\beta\ll\omega_o\ \rightarrow\ R\ll\sqrt{\frac{L}{C}}$, *That's why* $q_{max}=q_{rez}=\frac{e_o/L}{2\beta\sqrt{-\beta^2+\omega_o^2}}\approx\frac{e_o}{R\omega_o}$.

The obtained result can be interpreted as follows: the closer the EMF frequency is to the resonant frequency of the circuit, the greater the maximum charge appears on the capacitor plates [6]:



4-rast. Resonance hodissining graphics

From this graph it can be seen that true resonance is observed at β=0. When we compare electromagnetic resonance with mechanical ones, then it is clear that in electromagnetic oscillatory processes, voltage resonance on the capacitance, current resonance, and voltage resonance on the inductance are observed, the conditions of which are the following [7]:

$$\omega_C(rez) = \sqrt{\omega_0^2 - 2\beta^2} = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}};\ I_{rez} = \frac{e_0}{R};\ \omega_L(rez) = \frac{1}{\sqrt{LC - \frac{R^2C^2}{2}}};$$

The quality factor of the system, expressed through resonant characteristics. Let the natural frequency $\omega_0 \approx \omega\_св$ -frequency of free damped oscillations. For electromagnetic oscillatory processes, the quality factor is numerically equal to the ratio $Q = \frac{U_C(rez)}{e_0}$.

As a conclusion, we can say that resonance in electromagnetic oscillatory circuits is used in the transmission and reception of electromagnetic waves. Since communication is mainly carried out through the air using electromagnetic waves. Resonance in oscillatory circuits is also used in the military sphere as radio warfare.

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