Proceedings of International Conference on Scientific Research in Natural and Social Sciences
Hosted online from Toronto, Canada.
Date: 5th October, 2023
ISSN: 2835-5326 **Website:** econferenceseries.com

THE INVERSE PROBLEM OF FINDING THE RETARDATION FACTOR a_2 AND THE DIFFUSION COEFFICIENT D IN A SUBSTANCE TRANSPORT EQUATION IN A HOMOGENEOUS POROUS MEDIUM

T.O.Dzhiyanov, M.H.Turayev, R.U.Karshiyeva.

Samarkand State University, Samarkand, Uzbekistan

The equations of solute transport in the one-dimensional case are written in the form

$$\rho\frac{\partial S_{a1}}{\partial t}+\rho\frac{\partial S_{s1}}{\partial t}+\theta_1\frac{\partial c_1}{\partial t}+a_2\frac{\partial^\gamma c_1}{\partial t}=\theta_1 D_1\frac{\partial^2 c_1}{\partial x^2}-\theta_1 v_1\frac{\partial c_1}{\partial x}, \quad (1)$$

where a_2 – is the coefficient due to the solute transport into the second medium, 〖s〗^(β-1), γ-is the order of the derivative.

Let the problems of finding coefficients a_2 and D_1 from (1) be given.

The sedimentation of matter in each of the sections of the zones occurs reversibly in accordance with the kinetic equations

$$\rho\frac{\partial S_{a1}}{\partial t}=\theta_1 k_{a1}c_1-\rho k_{ad1}S_{a1}, \quad (2)$$

$$\rho\frac{\partial S_{s1}}{\partial t}=\theta_1 k_{s1}c_1-\rho k_{sd1}S_{s1}, \quad (3)$$

where k_a1, k_s1 – coefficients of deposition of matter from the fluid phase to the solid phase, s^(-1); k_ad1, k_sd1-the coefficients of separation of the substance from the solid phase and the transition to the fluid, s^(-1).

Let a fluid with a constant concentration of a substance be pumped into a medium initially saturated with a pure (without substance) fluid from the initial moment of

ISSN: 2835-5326 Website: econferenceseries.com

time c_0. Let us consider such periods of time where the concentration field does not reach the right boundary of the medium x→∞. Under these assumptions, the initial and boundary conditions for the problem have the form

$$c_1(0,x)=0,\ S_{a1}(0,x)=0,\ S_{s1}(0,x)=0, \quad (4)$$

$$c_1(t,0)=c_0, \quad (5)$$

$$\frac{\partial c_1}{\partial x}(t,\infty)=0. \quad (6)$$

Problem (4) – (9) is solved by the finite difference method [21]. In the considered area Ω={(t,x),0≤t≤T,0≤x≤∞} introduced a grid uniform in directions

$$\overline{\omega_{\tau h}}=\left\{\begin{matrix}(t_j,x_i);\ t_j=\tau j,x_i=ih,\\ \tau=\frac{T}{J},i=\overline{0,I},j=\overline{0,J}\end{matrix}\right\},$$

where I-is a sufficiently large integer selected so that segment [0,x_I],x_I=ih, overlaps the area of the calculated change in fields c_1,S_a1 and 〖S〗_s1,h- the grid step in the direction of x, – is the grid step in time .

In the open grid area

$$\omega_{\tau h}=\left\{\begin{matrix}(t_j,x_i);\ t_j=\tau j,x_i=ih,\\ \tau=\frac{T}{J},i=\overline{1,I-1},j=\overline{1,J}\end{matrix}\right\}$$

equations (4), (5), (6) were approximated as follows

$$\rho\frac{(S_{a1})_i^{j+1}-(S_{a1})_i^j}{\tau}+\rho\frac{(S_{s1})_i^{j+1}-(S_{s1})_i^j}{\tau}+\theta_1\frac{(c_1)_i^{j+1}-(c_1)_i^j}{\tau}$$
$$+\frac{a_2\tau^{1-\gamma}}{\Gamma(2-\gamma)}\left[\sum_{k=0}^{j-1}\frac{(c_1)_i^{k+1}-(c_1)_i^k}{\tau}((j-k+1)^{1-\gamma}-(j-k)^{1-\gamma})\right.$$
$$\left.+\frac{((c_1)_i^{j+1}-(c_1)_i^j)\tau^{1-\gamma}}{\tau}\right]$$
$$=\theta_1D_1\frac{(c_1)_{i-1}^{j+1}-2(c_1)_i^{j+1}+(c_1)_{i+1}^{j+1}}{h^2}$$
$$-\theta_1v_1\frac{(c_1)_i^{j+1}-(c_1)_{i-1}^{j+1}}{h} \qquad (7)$$

$$\rho\frac{(S_{a1})_i^{j+1}-(S_{a1})_i^j}{\tau}=\theta_1k_{a1}(c_1\ )_i^j-\rho k_{ad1}(S_{a1})_i^{j+1}, \qquad (8)$$

$$\rho\frac{(S_{s1})_i^{j+1}-(S_{s1})_i^j}{\tau}=\theta_1k_{s1}(c_1\ )_i^j-\rho k_{sd1}(S_{s1})_i^{j+1}, \qquad (9)$$

where $(c_1)_i^j, (S_{a1})_i^j, (S_{s1})_i^j$ – grid function values $c_1(1,x),\ S_{a1}(t,x),\ S_{s1}(t,x)$, at the point (t_j, x_i).

From explicit grid equations (11), (12) we determine $(S_{a1})_i^{j+1}$, $(S_{s1})_i^{j+1}$

$$(S_{a1})_i^{j+1}=p_{b1}(S_{a1})_i^j+p_{b2}, \qquad (10)$$

$$(S_{s1})_i^{j+1}=q_{b1}(S_{s1})_i^j+q_{b2}, \qquad (11)$$

Where

$$p_{b1}=\frac{1}{1+\tau k_{ad1}},\quad p_{b2}=\frac{\tau\theta_1 k_{a1}}{\rho+\rho\tau k_{ad1}}(c_1)_i^j,$$

$$q_{b1}=\frac{1}{1+\tau k_{sd1}},\qquad q_{b2}=\frac{\tau\theta_1 k_{s1}}{\rho+\rho\tau k_{sd1}}(c_1)_i^j.$$

Grid equations (7) are reduced to the form

$$A_1(c_1)_{i-1}^{j+1}-B_1(c_1)_i^{j+1}+E_1(c_1)_{i+1}^{j+1}=-(F_1)_i^j, \tag{12}$$

where $A_1=\frac{\theta_1 D_1\tau}{h^2}+\frac{\theta_1 v_1\tau}{h}$, $B_1=\theta_1+\frac{2\theta_1 D_1\tau}{h^2}+\frac{\theta_1 v_1\tau}{h}+\frac{a_2\tau^{1-\gamma}}{\Gamma(2-\gamma)}$, $E_1=\frac{\theta_1 D_1\tau}{h^2}$,

$$(F_1)_i^j=\left(\theta_1+\frac{a_2\tau^{1-\gamma}}{\Gamma(2-\gamma)}\right)B_1(c_1)_i^j-\rho\left((S_{a1})_i^{j+1}-(S_{a1})_i^j\right)-\rho\left((S_{s1})_i^{j+1}-(S_{s1})_i^j\right)$$
$$-\frac{a_2\tau^{1-\gamma}}{\Gamma(2-\gamma)}\left[\sum_{k=0}^{j-1}((j-k+1)^{1-\gamma}-(j-k)^{1-\gamma})(c_1)_i^{k+1}\right.$$
$$\left.-((j-k+1)^{1-\gamma}-(j-k)^{1-\gamma})(c_1)_i^k\right].$$

$$I_1=\alpha(C_2-C_1),\qquad I_2=a_2\frac{\partial^\gamma C_1}{\partial t^\gamma}.$$

The following procedure for calculating the solution is established. By (10), (11) (S_a1)_i^(j+1), (S_s1)_i^(j+1), are determined, then the system of linear equations (12) is solved by the Thomas' algorithm with respect to (c_1)_i^(j+1). Since p_b1,q_b1<1, schemes (10), (11) are stable, and for (12) the stability conditions of the Thomas' algorithm are satisfied.

we minimize the function using the above, Values describe the standard deviation for the entire time period. I_1 and I_2 the performed calculations show that the minimum value of 1 is reached

The closeness of the flow times should guarantee the closeness of the concentration fields determined using the proposed approach . I_1 and I_2 and the model [4]. To quantitatively estimate their closeness, we use the standard deviation of type (14), which is determined based on only two models, i.e.

$$F(a_2,\gamma) = \int_0^T\int_0^L \left(C_1^{(1)} - C_1^{(2)}\right)^2 dxdt,$$

where C_1^((1)) is the concentration field C_1 (t,x) at a given t, defined according to [4], and C_1^((2)) is the same here. The following minimum value was obtained for the cases analyzed above

$$F(a_2, D) = 0{,}002347387654452 \ \ for \ \ a_2 = 0{,}0006, D = 10^{-6}.$$

The inverse problem of finding a_2 retardation factor and D diffusion coefficient was numerically solved in the given model of substance transport.

REFERENCES

1. Barenblatt G.I., Entov V.M. and Ryzhik V.M. Theory of Fluid Flow Through Natural Rocks. Kluwer Academic, Dordrecht, The Netherlands. 1990.
2. Van Golf-Racht T.D. Fundamentals of Fractured Reservoir Engineering. Developments in Petroleum Science, Vol. 12. Elsevier. 1982 y. 732 p.
3. Sahimi M. Flow and Transport in Porous Media and Fractured Rock. From Classical Methods to Modern Approaches. Second, Revised and Enlarged Edition. WILEY-VCH VerlagGmbH&Co. KGaA. 2011.
4. Leij F.L., Bradford S.A. Colloid transport in dual-permeability media // Journal of Contaminant Hydrology. 150.- 2013.-P. 65–76.
5. Khuzhayorov B. Kh, Djiyanov T.O. Solute Transport with nonequilibrium adsorption in an inhomogeneous porous medium // Scientific journal «problems of computational and applied mathematics». -2017. -№ 3(9). – P. 63-70.
6. Samarskiy A.A. The theory of difference scheme. M. The science. 1977. P. 656.

7. Cey E.E., Rudolph D.L. Field study of macropore flow processes using tension infiltration of a dye tracer in partially saturated soils // Hydrological Processes. 23.- 2009.-P. 1768–1779.
8. Jarvis N.J. A review of non-equilibrium water flow and solute transport in soil macropores: principles, controlling factors and consequences for water quality // European Journal of Soil Science. 58.- 2007.- P. 523–546.
9. Pang L., McLeod M., Aislabie J., Simunek J., Close M., Hector R. Modeling transport of microbes in ten undisturbed soils under effluentirrigation // Vadose Zone Journal 7. -2008.- P. 97–111.
10. Passmore J.M., Rudolph D.L., Mesquita M.M.F., Cey E.E., Emelko M.B. The utility of microspheres as surrogates for the transport of E. coli RS2gin partially saturated agricultural soil // Water Research 44.-2010.- P. 1235–1245.
11. Gerke H.H., van Genuchten M.T. Macroscopic representation of structural geometry for simulating water and solute movement in dual porosity media // Advances in Water Resources. 19. -1996. -P. 343–357.
12. Simunek J., van Genuchten M.Th. Modeling nonequilibrium flow andtransport processes using HYDRUS //Vadose Zone Journal 7.- 2008. -P. 782–797.
13. Van Genuchten M.Th., Wierenga P.J. Mass Transfer Studies in Sorbing Porous media.I. Analytical Solution // Soil Science Society of America Journal, 1976.- Vol.40, N.4.- P.473−480.
14. Bradford S.A., Simunek J., Bettahar M., van Genuchten M.T., Yates S.R. Modeling colloid attachment, straining, and exclusion in saturated porous media // Environmental Science & Technology. 37.-2003.- P. 2242–2250.
15. Bradford S.A., Torkzaban S. Colloid transport and retention inunsaturated porous media: A review of interface-, collector-, and pore-scale processes and models //Vadose Zone Journal. 7.- 2008.- P. 667–681.
16. Elimelech M. et al. Particle Deposition and Aggregation: Measurement, Modelling, and Simulation. Butterworth-Heinemann.- Oxford, England,1995.
17. Gitis V., Rubinstein I., Livshits M.,Ziskind M. Deep-bed filtration model with multistage deposition kinetics // Chemical Engineering Journal. 163.- 2010.- P. 78-85.