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ON A PROBLEM FOR A QUASI-LINEAR ELLIPTIC EQUATION WITH TWO PERPENDICULAR LINES OF DEGENERACY

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The study of quasi-linear equations of elliptic, hyperbolic and mixed types is important because they are interesting both theoretically and practically. Boundary value problems for such equations with one line of degeneracy are studied in [1-2]. However, boundary value problems for equations with two lines of degeneracy are poorly studied. Note the works [3-5]. This paper is devoted to the study of a boundary value problem for a quasi-linear elliptic equation with two degeneracy lines.

Consider the equation

$$y^m u_{xx} + x^m u_{yy} + C(x,y)u = f(x,y,u), \; m = const > 0. \qquad (1)$$

Пусть Ω - конечная односвязная область, ограниченная гладкой кривой σ с концами в точках A$(1,0)$ B$(0,1)$ и отрезками OA: $y = 0$ оси и OB: $x = 0$.

Let Ω be a finite simply connected region bounded by a smooth curve σ with ends at points A$(1,0)$ B$(0,1)$ and segments OA: $y = 0$ of the axis and OV: $x = 0$.

Let 's introduce the notation:

P=$\{(x,y)(x,y) \in \Omega, \; -\infty < u < +\infty\}$,

$I_1 = \{(x,y) : 0 \le x \le 1, \; y = 0\}, \; I_2 = \{(x,y) \; : 0 \le y \le 1, \; x = 0\}$,

$2p = m + 2, 2\beta = m/(m+2)$.

Further, with respect to the curve σ, we will assume that:

1) let the parametric equations of the curve σ be $x = x(s), y = y(s)$;

2) the functions $x(s)$ and $y(s)$ have continuous derivatives $x'(s)$and $y'(s)$ on the segment $[0,l]$ that do not simultaneously vanish, the derivatives $x''(s)$ and $x''(s)$ satisfy the Helder condition of orderλ_0 $(0 < \lambda_0 < 1)$ on $[0,l]$, where l is the arc length calculated from point A(1,0);

3) around the points $A(1,0)$ and $B(0,1)$, the following conditions are met

$$x^{\frac{m}{2}} \left|\frac{dx}{ds}\right| \le const \; y^{m+1}(s), \qquad y^{\frac{m}{2}} \left|\frac{dy}{ds}\right| \le const \; x^{m+1}(s),$$

and here $x(1) = y(0) = 1,\ x(0) = y(1) = 1$.

Definition: A regular solution of equation (1) in a domain Ω is a function $u(x,y) \in C(\bar{\Omega}) \cap \mathrm{C}^1(\Omega\ \cup \sigma) \cap C^2(D)$, satisfying equation (1) in Ω , which has a bounded second derivative in $\partial\Omega$, except for the points O(0,0) and A(1,0), B(0,1), where they can go to infinity of order less than one and λ_1, respectively, where λ_1 is sufficient a small positive number and $0 < \lambda_1 < \lambda$, where λ is given in the continuation of the article.

Problem. Find a function $u(x,y)$ from the class $C(\bar{\Omega}) \cap \mathrm{C}^1(\Omega\ \cup \sigma) \cap C^2(D)$, satisfying the boundary conditions

$$A_s[u]|_\sigma = \varphi(s), 0 < s < l,$$
$$u(x,y)|_{\mathrm{OA}} = \psi(x), x \in I_1,$$
$$u(x,y)|_{\mathrm{Ob}} = g(y), y \in I_2,$$

where $\varphi(s), \psi(x), g(y)$ are given sufficiently smooth functions, and $\psi(0) = g(0)$, $A_s[u] = y^m \frac{dy}{ds}\frac{\partial u}{\partial x} - x^m \frac{dx}{ds}\frac{\partial u}{\partial y}$.

We will assume that the right side of equation (1) satisfies the condition

$$f(x,y,u) = (xy)^m f_1(x,y,u),$$

where the function $f_1(x,y,u)$ is continuous and has continuous first-order derivatives with respect to all arguments in P and vanishes on σ of order $1+\lambda$, where λ is a sufficiently small number (which was introduced at the beginning of the article) and

$$\max_P\{|f_1|, |f_{1u}|\} \le const.$$

In this communication, under certain restrictions on the given functions, the unique solvability of the problem under study is proved.

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