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CLASSIC CONDENSER CAPACITY AND A NEW CAPACITY IN THE CLASS OF $m-sh$

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Annotation:
In this work we will construct connection between classic condenser capacity and a new capacity in the class of $m-sh$ functions.

Keywords: m-subharmonic functions, Laplace operator, capacity, condenser capacity, external capacity, m-convex domain, m-capacity.

Capacity is a set function from potential theory. Using Laplace operator $\Delta u = dd^c u \wedge \beta^{n-1}$ more comfortable and one of reasons of it this operator is linear operator. There are several important results obtained in works of Abdullaev B.[1], Sadulleav A.(see [2], [3], [9], [14]), Rakhimov K.(see [4], [9]), Bedford E. (see [5]), Brelo M. (see [7]), Landkof N. S. (see [8]), Aupetit B., Wermer J. (see [10]), Alexander H.(see [12], [13]), Taylor B.A. (see [11], [12]) and other mathematicians. For example, in work of Sadulleav A. [3] finding simple solution to "Is local pluripolar subset $E \subset \mathbb{C}^n$ pluripolar in $\mathbb{C}^n$?". That is one of Lelonne's problem and solution of this opens way for more results in potential theory. One of these kinds of important results, connection between different capacities. For the theorem firstly we need definitions of classic capacity and capacity in class of m-subharmonic functions.

Definition. [1] Let K compact in $D \subset \mathbb{C}^n$. The value

$$C_m(K,D) = \inf\left\{ \int_D (dd^c u)^m \wedge \beta^{n-m} : u \in m-sh(D) \cap C(D), \right.$$
$$\left. u|_K \leq -1, \varliminf_{z\to \delta D} u(z) \geq 0 \right\}$$

is called the condenser capacity (m-capacity) of (K, D).

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External capacity for classic capacity was defined in

$$C^*_m(E, D) = \inf\{C_m(U): U \supset E - \text{open set}\}$$

where capacity of the open set $C_m(U) = \sup\{C_m(K): K \subset\subset U\}$.
We will suppose that $D \subset \mathbb{C}^n$ is strongly m-convex domain, i.e. $D = \{\rho(z) < 0\}$, $\rho(z) \in m - sh(\overline{D})$. For a compact $K \subset\subset D$ we will define a class of functions

$$\mathcal{U}_m(K, D) = \left\{u(z) \in m - sh(D) \cap C(D): u|_K \leq -1, \varliminf_{x \to \partial D} u(z) \geq 0\right\} \quad \text{(see [2])}$$

and a capacity

$$\Delta_m(K) = \Delta_m(K, D) = \inf\left\{\int_D \Delta u : u \in \mathcal{U}_m(K, D)\right\}.$$

Definition. The external capacity of the set $E \subset D$ is defined by

$$\Delta^*_m(E) = \inf\{\Delta_m(U): U \supset E - \text{open set}\},$$

were

$$\Delta_m(U) = \sup\{\Delta_m(K): K \subset\subset U\}.$$

The next theorem about comparison of classic external capacity for condenser capacity and the external capacity Δ^*_m.

Theorem. Let $E \subset D$, then there exists a constant $M(D) > 0$ (depending on measure of D) such that

$$\sqrt[m]{C^*_m(E, D)} \leq M(D) \cdot \Delta^*_m(E, D)$$

This theorem is analog of theorems from works of A.Sadullaev (see [3]), Rakhimov K. (see [4]), Abdullaev B.I. (see [1]).

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