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SOME APPLICATIONS OF THE DEFINITE INTEGRAL

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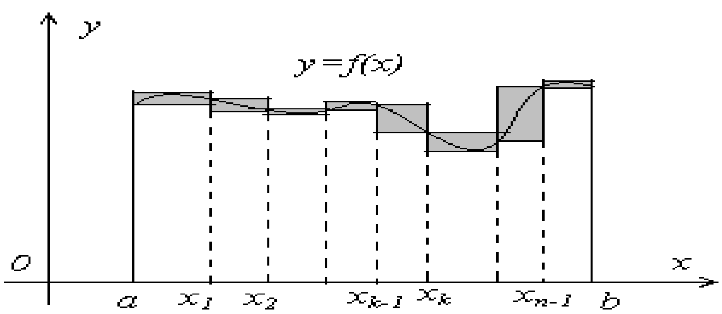
Abstract:
The article discusses some applications of the definite integral. First, the formula for calculating the surface of shapes using the exact integral is derived, and then it is strengthened by an example. Then the formula for calculating the face of a curve in a curved trapezoid given by parametric method is found and examples are also given. In addition to the solution of the examples, brief historical information about the forms is given.

Keywords: straight line, rectangle, non-negative, continuous, function, ellipse, cycloid, parametric equation

Surface calculation formulas. Let us suppose that a flat figure D bounded by the straight lines $x=a$, $x=b$, $y=0$ and the graph of the non-negative continuous function $y=f(x)$ is given. We calculate the face of this figure. For this, we take some τn division of the section $[a;b]$:

$$a=x_0<x_1<...<x_n=b.$$

Let the minimum and maximum values of f(x) on the section $[x_{k-1},x_k]$ be mk and Mk, respectively. We make two rectangles corresponding to each $[x_{k-1},x_k]$, whose base consists of this section, and whose heights are $y=m_k$ and $y=M_k$. A polygon consisting of all the smaller rectangles (heights μ) is an inscribed polygon, and a polygon consisting of large rectangles is an inscribed polygon. Their faces match



$$\sigma=\sum_{k=1}^{n} m_k \Delta x_k=\underline{S}(\tau_n), \quad \sigma'=\sum_{k=1}^{n} M_k \Delta x_k=\overline{S}(\tau_n)$$

will be. According to the condition, the function f(x) is continuous, from which it follows that it is integrable. So,

$$\sup\sigma=\lim_{\lambda\to 0}\underline{S}(\tau_n)=\lim_{\lambda\to 0}\overline{S}(\tau_n)=\inf\sigma' \quad (\lambda=\max_{1\le k\le n}\Delta x_k)$$

i.e. figure D (curved trapezoid) is squared and its face

$$S=\int_a^b f(x)dx$$

will be. If the figure D above is bounded by a line instead of the straight line y=0 below, and the function is continuous, then

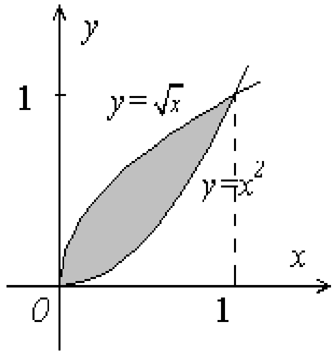
$$S=\int_a^b (f(x)-\varphi(x))dx$$

will be.

An example. Find the face of the figure bounded by the lines $y=x^2$ and $x=y^2$

Solving. The given figure is bounded from the top by the line $y=\sqrt{x}, \quad 0\le x\le 1$, and from the bottom by the line $y=x^2$. That is why

$$S=\int_0^1(\sqrt{x}-x^2)dx=\frac{2x^{\frac{3}{2}}}{3}\Bigg|_0^1-\frac{1}{3}x^3\Bigg|_0^1=\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$$



The curve in the curved trapezoid is a parametric method

$$\begin{cases} x=\varphi(t), \\ y=\psi(t) \end{cases}$$

$\alpha\le t\le\beta$ is given, where $\varphi(\alpha)=a$, $\varphi(\beta)=b$, $[\alpha;\beta]$ in section $\psi(t)$ is continuous, and $\varphi(t)$ is monotone and continuous $\varphi'(t)$. Based on the variable substitution rule, we have the following:

$$S=\int_a^b f(x)dx=\int_\alpha^\beta \psi(t)\varphi'(t)dt \qquad (1)$$

Calculate the face of the figure bounded by the axis Ox and one arch of the cycloid:

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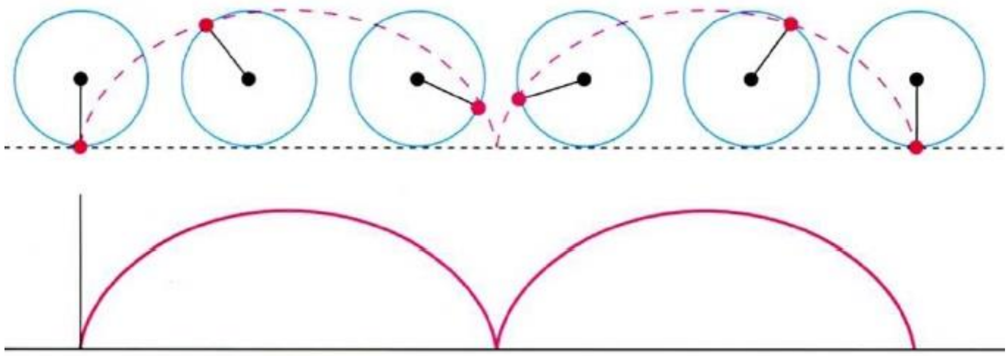
$$\begin{cases} x = a(t - \sin t), \\ y = a(1 - \cos t), \end{cases} \quad 0 \le t \le 2\pi$$

(Cycloids are defined as the trajectory of a specific point on a circle moving in a straight line. Cycloids have been studied by many famous mathematicians. One of the first scientists to examine these curves in detail was the famous scientist, the father of modern physics. The dead is Galileo Galilei (1564-1642). However, it is difficult to say that Galileo was lucky in this regard. In particular, he tried many times to calculate the surface bounded by a cycloid and a plane, but he did not succeed. He even made a similar curved shape from a metal plate. , who tried to calculate its surface by purely physical measurements, but still failed to achieve his goal.Rene Descartes (1598-1650) was the first to accurately calculate the surface of this figure.



It is equal to 3πr2, where r is the radius of the circle drawing the cycloid. After Descartes, Gilles Roberval (1602-1675) calculated the length of the arc drawn by this curve. This arc is also represented by a very simple mathematical formula: L=8a. (This curve also served to solve two important problems that have been a stumbling block for many scientists and engineers for many years.)
Solving. According to the formula (1):

$$S = \int_0^{2\pi} a(1-\cos t)a(1-\cos t)dt = a^2 \int_0^{2\pi} (1-\cos t)^2 dt =$$

$$= a^2 \left(\int_0^{2\pi} dt - 2\int_0^{2\pi} \cos t dt + \int_0^{2\pi} \cos^2 t dt \right) = a^2 \left((t - 2\sin t)\Big|_0^{2\pi} + \right.$$

$$\left. + \frac{1}{2}\int_0^{2\pi} (1+\cos 2t)dt \right) = a^2 \left(2\pi + \frac{1}{2}\left(t + \frac{1}{2}\sin 2t\right)\Big|_0^{2\pi} \right) = 3\pi a^2.$$

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