

THE ROLE OF DIFFERENTIAL EQUATIONS IN SOLVING MILITARY PROBLEMS

Rakhmanov Istam Yarikulovich

Professor of the Department of Exact science of the Academy of the Armed Forces of the Republic of Uzbekistan

Abstract:

The article discusses the possibilities of applying differential equations in solving problems related to the military sphere. Also, great attention is paid to solving military-practical problems aimed at building a mathematical model using differential equations. Using differential equations, relationships are established between the curve and its slope, the path traveled and the speed of movement.

Keywords: Differential equation, mathematical model of military-practical problems, aerodynamics, military sphere, optimal solution.

Mathematics, as a fundamental science, is used in almost all areas of social life. Mathematics is very necessary for modern society, because we are surrounded by computers and numbers on all sides. With the help of mathematics, it is possible to study texts, extract information and analyze it in terms of meaning. The application of mathematics to military needs was already known to the Babylonians. Therefore, the military sphere also played a significant role in the development of modern mathematics. The ancient Greek scientist Archimedes (c. 287 - 212 BC) made a great contribution to the formation of mathematics. He played a major role in the formation of mathematics by using his knowledge of mechanics, physics and the military sphere to solve practical problems using mathematics. At the beginning of the 20th century, the development of the scientific theory of the use of mathematics in aerodynamics by M.B. Keldyshev and his students solved the problem of aircraft failure due to vibration. The creation of a complex mathematical theory of flutter provided reliable protection against the occurrence of vibrations in aircraft.

Differential equations are a branch of mathematics that studies the theory and methods of solving equations involving an arbitrary variable, its function, and derivatives of various orders of this function (ordinary differential equations) or equations involving functions of several variables and their derivatives of various orders of this function (partial differential equations). A differential equation

expresses the relationship between an unknown function and its derivatives. Such a relationship is widely used in mechanics, physics, chemistry, biology, economics, and other fields.

When solving military-practical problems, first of all, its mathematical model is built, often this model consists of ordinary differential equations. These equations, which connect the arbitrary variable, the desired function, and its derivatives, are the basis of many laws of the material world. With the help of such equations, it is possible to establish connections between a curve and its slope, the path traveled by a moving body and its speed, as well as to express such well-known laws as Newton's second law and Hooke's law. Often, the process of constructing a differential equation expressing such connections is a complex mathematical problem.

Firstly, in order to construct a mathematical model of the phenomenon or process under consideration, in-depth knowledge of the relevant branches of science, such as physics, theoretical mechanics, and flight dynamics, is required. Secondly, the differential equation obtained as a result of constructing a mathematical model of the phenomenon or process under consideration should be reduced to a differential equation of a certain type as much as possible: linear, homogeneous, etc. Therefore, it is often necessary to use various simplifications, but at the same time it is necessary to take into account all the main factors affecting the process.

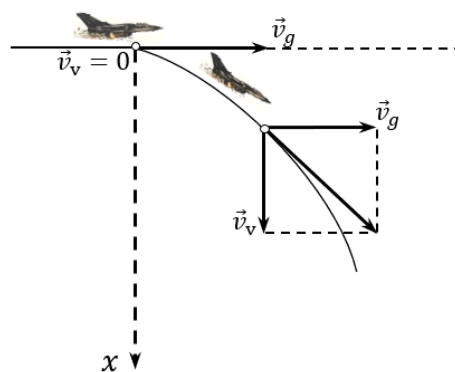
Let's consider a first-order differential equation and the problems of military practical significance that are brought to its solution.

Problem 1. A fighter jet flying horizontally dives. Determine the law of variation of the fighter jet's vertical dive velocity with respect to the distance flown. Assume that air resistance is proportional to the square of the velocity.

Solution.

When the plane is diving, it is affected by the force of gravity $P = mg$, the force of air resistance kV_H^2 (where H - the vertical distance traveled by the plane during time t).

Based on Newton's second law, we have the following differential equation:



$$m \frac{dv_H}{dt} = mg - k\mathcal{V}_H^2 \quad (1)$$

Since in the problem we need to determine the relationship between the velocity $\mathcal{V}_H$ and the distance traveled in the vertical direction H , we introduce the variable dH .

Then

$$\frac{dv_H}{dt} = \frac{dv_H}{dH} \cdot \frac{dH}{dt} = \frac{dv_H}{dH} \cdot \mathcal{V}_H.$$

Putting these findings into equation (1), we obtain the following equation:

$$m \cdot \frac{dv_H}{dH} \mathcal{V}_H = mg - k\mathcal{V}_H^2 \text{ or } \frac{mv_H dv_H}{mg - k\mathcal{V}_H^2} = dH,$$

from this

$$H = -\frac{m}{2k} \ln C (mg - k\mathcal{V}_H^2).$$

Here we find the value of the constant C using the initial condition $t = 0, H = 0, \mathcal{V}_H = 0$:

$$\frac{m}{2k} \ln C mg = 0 \Rightarrow C mg = 1 \Rightarrow C = \frac{1}{mg}$$

By substituting the value of this found constant C into the equation above, we find

$$H = -\frac{m}{2k} \ln \left(1 - \frac{k\mathcal{V}_H^2}{mg} \right)$$

This represents the law of variation of the vertical dive speed of an aircraft relative to the distance flown.

Problem 2. At an altitude of 2 km, an airplane is performing a combat turn, with a constant speed $v = 1080 \text{ km/s}$ and a trajectory angle of $\theta = 30^\circ$. How long will it take for the airplane to climb 3 km? How high will the airplane climb in 30 seconds?

Solution.

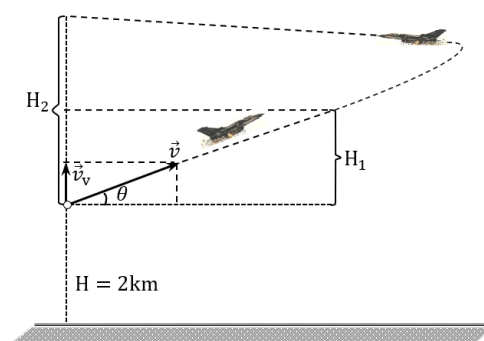
Let the height of the plane be H .

From the condition of the problem, we write

$$dH = dS \sin \theta.$$

$$\text{Then } \frac{dH}{dt} = \frac{dS}{dt} \sin \theta = v \sin \theta, \text{ hence } dt = \frac{dH}{v \sin \theta}.$$

Considering $v = 1080 \text{ km/h} = 300 \text{ m/second}$, we find from the last formula how long it will take the plane to rise 3 km:



$$t = \int_{2000}^{3000} \frac{dH}{v \sin \theta} = \frac{1}{v \sin \theta} H \Big|_{2000}^{3000} = \frac{2}{300} (3000 - 2000) \approx 6,7s$$

In a similar way, we find $dH = v \sin \theta dt$ and from this we find how high the plane will rise in 30 seconds:

$$H = \int_0^{30} v \sin \theta dt = v \sin \theta t \Big|_0^{30} = 300 \cdot \frac{1}{2} \cdot 30 = 4500 \text{ m} = 4,5 \text{ km}.$$

In conclusion, it can be said that mathematics in the military field helps to understand the essence of armed conflicts and find optimal solutions to military operations.

Used literature

1. Raxmonov I.Y THE ROLE OF MATHEMATICS IN THE DEVELOPMENT OF STUDENTS' THINKING // European Journal of Emerging Technology and Discoveries ISSN (E): 2938-3617 Volume 1, Issue 9, December, 2023
2. Raxmonov I.Y., Xalilayev B.T. Ofitserlar kasbiy faoliyatida kompetentlik va uning ahamiyati. "Kasb-hunar ta'limi" ilmiy-uslubiy, amaliy, ma'rifiy jurnali. №3, 2023.
3. Gulay T.A., Dolgopolova A.F., Litvin D.B., Meleshko S.V. Teoriya veroyatnostey i matematicheskaya statistika / Uchebnoe posobie / Stavropol, 2013
4. Filippov A. F. Sbornik zadach po differentialnym uravneniyam.-Izhevsk: NITs "Regularnaya i chaotic dynamics", 2000. -176p.